

Question #3

Given that, $W = \{ \langle 1, -1, 1 \rangle, \langle 0, 1, 2 \rangle \}$ and $V = \langle 2, -3, 4 \rangle$

As $\langle 1, -1, 1 \rangle$ is not a scalar multiple of $\langle 0, 1, 2 \rangle$, $\langle 1, -1, 1 \rangle$ and $\langle 0, 1, 2 \rangle$ are linearly independent.

$\therefore \{ \langle 1, -1, 1 \rangle, \langle 0, 1, 2 \rangle \}$ is a basis for W .

$$\begin{aligned} \text{Let } v_1 &= \langle 1, -1, 1 \rangle, \quad v_2 = \langle 0, 1, 2 \rangle - \frac{\langle 0, 1, 2 \rangle \cdot \langle 1, -1, 1 \rangle}{\langle 1, -1, 1 \rangle \cdot \langle 1, -1, 1 \rangle} \langle 1, -1, 1 \rangle \\ &= \langle 0, 1, 2 \rangle - \frac{1}{3} \langle 1, -1, 1 \rangle = \langle -\frac{1}{3}, \frac{4}{3}, \frac{5}{3} \rangle \end{aligned}$$

then $\{v_1, v_2\}$ is an orthogonal basis for W .

$$\begin{aligned} \text{Define } w_1 &= \frac{v \cdot v_1}{v_1 \cdot v_1} v_1 + \frac{v \cdot v_2}{v_2 \cdot v_2} v_2 \\ &= \frac{\langle 1, -1, 1 \rangle \cdot \langle 2, -3, 4 \rangle}{\langle 1, -1, 1 \rangle \cdot \langle 1, -1, 1 \rangle} \langle 1, -1, 1 \rangle + \frac{\langle 2, -3, 4 \rangle \cdot \langle -\frac{1}{3}, \frac{4}{3}, \frac{5}{3} \rangle}{\langle -\frac{1}{3}, \frac{4}{3}, \frac{5}{3} \rangle \cdot \langle -\frac{1}{3}, \frac{4}{3}, \frac{5}{3} \rangle} \langle -\frac{1}{3}, \frac{4}{3}, \frac{5}{3} \rangle \\ &= 3 \langle 1, -1, 1 \rangle + \frac{1}{7} \langle -1, 4, 5 \rangle \\ &= \frac{1}{7} \langle 20, -17, 26 \rangle \end{aligned}$$

$$\begin{aligned} \text{and } w_2 &= v - w_1 = \langle 2, -3, 4 \rangle - \frac{1}{7} \langle 20, -17, 26 \rangle \\ &= \frac{1}{7} \langle -6, -4, 2 \rangle \end{aligned}$$

Then $V = w_1 + w_2$, where $w_1 \in W$ and $w_2 \in W^\perp$